Vehicle Crash Reconstruction with Uncertain Input Parameters

W. Hugemann, H. Maurer

1 Problem Statement

Fig. 1 shows a typical traffic accident where $\vec{v}_1{}'$ and $\vec{v}_2{}'$ are the post-crash velocities immediately after the collision and $\vec{v}_1$ and $\vec{v}_2$ are the pre-crash velocities shortly before the collision. Reconstruction of a traffic accident for *forensic puposes* involves the task to determine the pre-crash velocities of the vehicles. The starting point for the reconstruction is always the situation after the occurance of the accident, i.e. the final positions of the vehicles, their deformations and the tire-marks on the road surface. An analysis of the post-crash-motion with respect to the dissipation of energy leads to the post-crash velocities $\vec{v}_1{}'$ and $\vec{v}_2{}'$. Based on this set of data we shall present an algorithm for computing the pre-crash velocities $\vec{v}_1$ and $\vec{v}_2$. Furthermore, the original velocities may be calculated allowing for possible braking-manoeuvres.

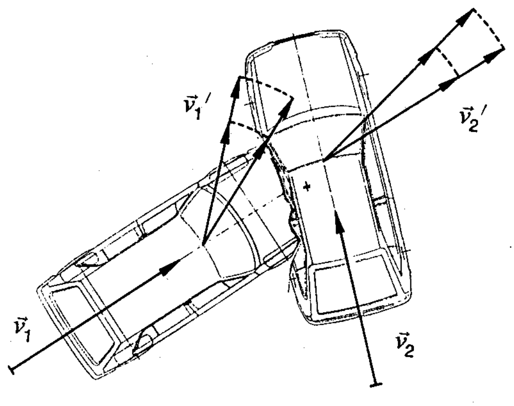

Fig.1: Two-dimensional collision

The analysis of post-crash motion is always afflicted with uncertainties. For example, the retardation of the vehicles after the collision can vary between a lower and upper limit, leading to corresponding variations of the post-crash velocities. The same applies to the direction of motion after the collision. Juridical considerations are (or at least should be) unaffected by probability-distributions, because the only question of interest is provable guilt or innocence. Hence there is no need to assume a priori any probability-distribution for the post-crash velocities (and further uncertain input parameters). Moreover the subsequent analysis of the collision may give evidence that the calculation of the post-crash-velocities was based on wrong assumptions, leading to contradictions.

A suitable algorithm for the calculation of the pre-crash velocities should therefore feature the following characteristics:

- it should be able to calculate the lower and upper limits of the pre-crash-velocities,
- it should give information about the interdependence of the pre-crash-velocities, because for physical reasons the calculated limits may not be combined arbitrarily,
- it should provide an indicator for possible contradictions resulting from the analysis of post-crash motion,
- it should run on a PC with respond times of a few seconds so that the input parameters of the analysis can be changed interactively.

Furthermore, the information should be displayed in such a manner that incorrect input is detected in any case.

2 Physical Basis

During the collision the motion of the vehicles is almost exclusively determined by forces acting between the body-works (and not by tire-forces, for instance). In principle, the motions — and thereby the post-crash-velocities — could be treated as an initial-value-problem of a differential equation. This would require a given force-law combining the distance between the cars and their relative orientation with the resulting repulsive force. Unfortunately, a realistic description of the repulsive force in such terms is far beyond todays technical standard. On the other hand, a simulation based on a realistic finite-element-model of the body-works would be far too complicated for the desired hardware-platform.

The analysis is therefore restricted to the application of principles which hold independently of the force-law [1]. These principles are the conservation of
linear momentum

(1) $$m_1 \vec{v}_1 + m_2 \vec{v}_2 = m_1 \vec{v}_1{}' + m_2 \vec{v}_2{}' ,$$

energy

(2) $$m_1 v_1^2 + m_2 v_2^2 = m_1 v_1'^2 + m_2 v_2'^2 + 2\,\Delta E$$

and angular momentum

(3) $$m_1 \vec{r}_1 \times \vec{v}_1 + m_2 \vec{r}_2 \times \vec{v}_2 = m_1 \vec{r}_1' \times \vec{v}_1' + m_2 \vec{r}_2' \times \vec{v}_2' + J_1\,\vec{\omega}_1' + J_2\,\vec{\omega}_2' ,$$

where m_i denote the masses of the cars, J_i their moments of inertia, $\vec{r}_i$ resp. $\vec{r}_i'$ denote the positions of the centers of gravity, $\vec{\omega}_i$ are the angular velocities and ΔE is the loss of kinetic energy due to the deformation of the vehicles. Also we have used the notation $v_i = |\vec{v}_i|$ and $v_i' = |\vec{v}_i'|$. Consequently, the application of these more general principles can not be as powerful as a description in terms of a force-law. It is for instance no longer possible to make use of additional information like the mutual orientation of the vehicles at the beginning of contact.

As the *direction* of pre-crash motion is generally rather precisely defined by the surrounding scene of the accident, we can restrict the analysis to the calculation of the *values* v_i of the pre-crash velocities.

3 Mathematical Treatment

3.1 The direct approach in the one-dimensional case

In an earlier paper [2] this problem was treated analytically for one-dimensional collisions where the pre- and post-crash velocities are considered as collinear. In this case the conservation of angular momentum supplies no additional information, because eqs. (1) and (3) are linear dependent (the post-crash angular velocities can be neglected under these circumstances). For given input parameters $\vec{p} = (v_1', v_2', \Delta E)$ eqs.

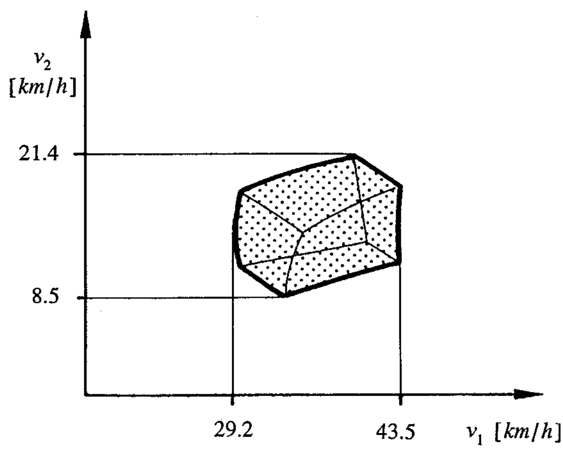

Fig.2: Solution set of an one-dimensional collision

(1) and (2) can be solved explicitly defining two solutions $\vec{x}(\vec{p}) \in \mathbb{R}^2$, which are both physically relevant. For a given accident one of the solutions can be excluded from the outset by an evaluation of the deformations of the vehicles. Then the solution of eqs. (1) and (2) determines a uniquely defined mapping $T_1: \vec{p} \Rightarrow \vec{x}(\vec{p})$. The lower and upper limits of the input parameter vector $\vec{p}$ define a cube $R \subset \mathbb{R}^3$. The image $T_1(R) \subset \mathbb{R}^2$ in the (v_1, v_2)-plane gives the solution set $S \subset \mathbb{R}^2$.

It has been shown in [2] that boundary points of the cube $R \subset \mathbb{R}^3$ are mapped into boundary points of the image $T_1(R) \subset \mathbb{R}^2$. Moreover, it can be argued that the boundary curve of the solution set in the (v_1, v_2)-plane is composed by the images of the *edges* of the cube R. Therefore it suffices to calculate the image of these edges, compare **Fig. 2**.

3.2 The Nonlinear-Least-Squares approach in the two-dimensional case

In case of a two-dimensional collision, the possible variations of the directions of post-crash motion have to be taken into account leading to a five-dimensional *input parameter vector* $\vec{p}$ whose lower and upper limits define a cube $R \subset \mathbb{R}^5$. As we are only interested in the values v_i of the pre-crash velocities, the set of eqs. (1) - (3) is now overdetermined, even if eq. (3) is omitted which is common practice. Note that eq. (1) is a vector equation, while eq. (3) is a scalar equation, because all vectors are collinear. Now we consider eqs. (1) - (3) in implicit form

(4) $$f_i(\vec{x},\vec{p}) = 0 \qquad , i = 1,2,3\ (,4)$$

where

(5) $$\vec{p} = (v_1', v_2', \Delta E, \varphi_1', \varphi_2') \in R \subset \mathbb{R}^5 .$$

Here φ_i' denote the the angles between the post-crash velocity vectors $\vec{v}_i'$ and the pre-crash velocity vector v_1. In order to obtain an approximate solution of (5) we consider the following *parametric nonlinear least-squares problem*

(6) $$\min_{\vec{x} \in \mathbb{R}^2} \sum_{i=1}^{3\,(4)} w_i\, f_i(\vec{x},\vec{p})^2$$

with prescribed weights $w_i > 0$. The solution of this optimisation problem is not difficult and can be obtained by classical *Newton-techniques*. For a given accident there is only one solution $\vec{x}(\vec{p}) \in \mathbb{R}^2$ which is of physical relevance. Though this solution can not be obtained analytically, the parametric problem (6) then defines a mapping T_2: $\vec{p} \Rightarrow \vec{x}(\vec{p})$. The application in practice supports the idea that this mapping T_2 has topological properties. To calculate the solution set$S = T_2(R) \subset \mathbb{R}^2$ we therefore can apply the same procedure as in the one-dimensional case: we combine the optimisation in (6) with an algorithm tracing the edges of the cube $R \subset \mathbb{R}^5$ with a prescribed step-size.

The system of eqs. (1) - (3), resp. eqs.(4), is rather simple, so that the first and second derivatives of the functions f_i in (6) may be calculated analytically for use in the numerical computation. Since the boundary curves in **Fig. 3** do not have strong curvature, it suffices to compute the solution $\vec{x}(\vec{p})$ for at most five mesh-points on every edge of the cube $R \subset \mathbb{R}^5$. As the cube $R \subset \mathbb{R}^5$ has 80 (= 5 x 2^4) edges, we have to solve (6) about 300 times which takes a few seconds on a 486-PC. Solving (6) we gain two indicators for possible contradictions in the input-parameter set. An inconsistent input vector will lead to a large residual in the numerical calculation. Indicating the magnitude of the residuals by colour, regions of inconsistent input vectors can be marked. Under these circumstances we would (for the same reason) also detect, that the shape of the solution set strongly depends on the choice of the weigths w_i.

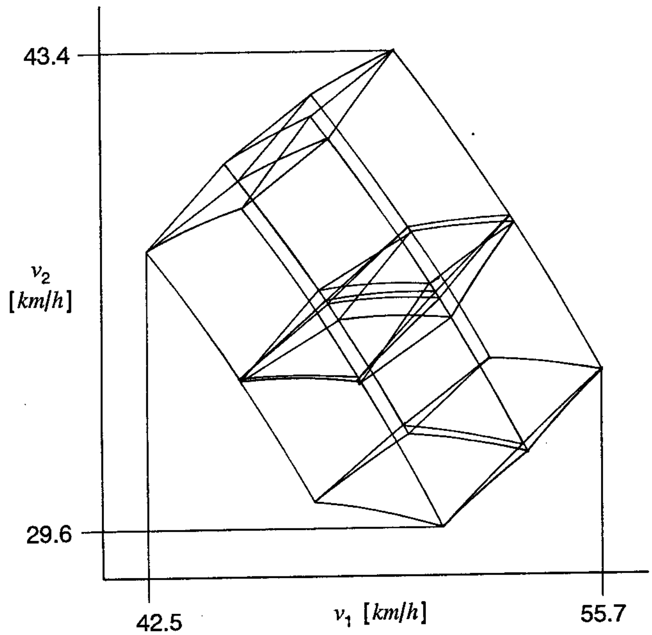

Fig.3: Solution set of a two-dimensional collision

Parameter set of the numerical example:

$$m_1 = 1540\,kg, \qquad v_1' = 35 - 40\ km/h, \qquad \varphi_1' = 15° \pm 2°,$$

$$m_2 = 1350\,kg, \qquad v_2' = 40 - 45\ km/h, \qquad \varphi_2' = 20° \pm 2°,$$

$$\Delta E = 25.0 - 44.4\,kNm, \qquad \varphi_2 = 45°,$$

$$w_1 = w_2 = w_3 = 1, \qquad w_4 = 0.$$

Fig.3 shows the shape of the solution set for the preceeding practical data and clearly illustrates the interdependence between the limits of the calculated velocities. For instance, the point combining the upper limits of both velocities does not belong to the solution set. The parameter set is choosen such that the solution set is nearly independent of the choice of the weights w_i.

References

[1] Hirschfelder, J.C.; Curtiss, C.F.; Bird, R.B.: Molecular Theory of Gases and Liquids. Wiley & Sons, New York 1964

[2] Hugemann, W.: Der eindimensionale Stoß als dreidimensionale Abbildung. Verkehrsunfall und Fahrzeugtechnik 30 (1992), S.103-107 u. 135-137

Addresses:

Dipl.-Ing. Wolfgang Hugemann
Ing.-Büro
Schimmelpfennig+Becke
Münsterstr. 101
D-48155 Münster

Prof. Dr. Helmut Maurer
Institut für Numerische und
Instrumentelle Mathematik,
Westfälische Wilhelms-Universität Münster
Einsteinstr. 62
D-48149 Münster